The Paper Dyno

You used to hear a lot of talk about *power:* horsepower, and plenty of it. These days, if you ask somebody how much power their car has, you're likely to get an answer like "150 watts per channel." A lot of cars are equipped with a motor that would be better suited for a vacuum cleaner but they've got a stereo amp that could weld deck plates on a battleship. But since you're reading this article, you're probably interested in power, namely horsepower, just like the people you're racing against. You know that power's good and more power's better, but there's a lot of confusion about exactly what it is.

Take for example that time you and your buddies were changing the fuel pump in Randy's Camaro. As usually happens when friends get together the discussion turns to horsepower. Everyone's talking about the astronomical amounts of power the engines had back in the sixties and someone mentions how engines are now rated in net instead of gross horsepower. Randy's about to get in a fight with Scott about whether a 350 Chevy puts out more horsepower than a 351 Cleveland. Then Dave, who's been talking about his classes down at the community college instead of getting his hands dirty, chimes in with " 'Course, everybody knows that *torque* accelerates, not horsepower."

"Oh, yeah, yeah, sure, everybody knows that," the collective assembly mutters. And you're all thinking "Wow. That sounds really technical. All this time, we've been talking about horsepower and now the in thing is torque."

So is power not important, or what? Saying that horsepower isn't what accelerates a car is true, but misleading, because it implies that torque and horsepower are independent, which is certainly not true. The intent of The Paper Dyno is to give you a tool by which you can plot a curve of the torque and power your car is putting out, but first we have to sort out the mystical relationship between torque, horsepower, and acceleration.

Conceptual definitions (For the right side of your brain)

The most basic definition of horsepower is that it is the *rate* that torque is being output. Let's start with an example to calibrate our thinking. Think back to the fuel pump we changed in Randy's Camaro. If we put a pressure gauge in the fuel line, we see an interesting phenomenon. When the needle valve in Randy's double pumper is closed the pump is developing high pressure in the lines. Then, at higher flows, the pressure starts to go down (**Figure 1**). As the flow increases, it will eventually outrun the pump's capability to increase pressure. A pump that is able to maintain a higher pressure at a high flow is said to be more *powerful*. Hold that thought while we apply that same principle to a moving car.

When you had to push Randy's car home with a bad fuel pump, you applied a force to the car. When the car was completely stopped with the brakes applied you could exert a certain force on his car. If you could put a force sensor (like a massive bathroom scale) between the bumpers you could have measured that force (**Figure 1**). As the car begins to move you'll be able to exert a good amount of thrust for a while, but as the car moves faster and faster the force will get less and less. Now think about that a minute. A more powerful engine will be able to keep the numbers on the bathroom scale high at a faster speed. That's what power is: a measure of the rate that force is applied. Think of torque as a particular kind of force and it should start to make sense.

Here's one more quick example to help us visualize this thing. You've probably seen wheelstand racers like Ed Jones's Jolly Rancher fire engine that pull the front wheels and keep them up all the way down the track. What raises them up into a wheelie is the physical quantity torque; the continued application of that torque that keeps the wheels up while they accelerate down the track is a *measurement* we call horsepower.

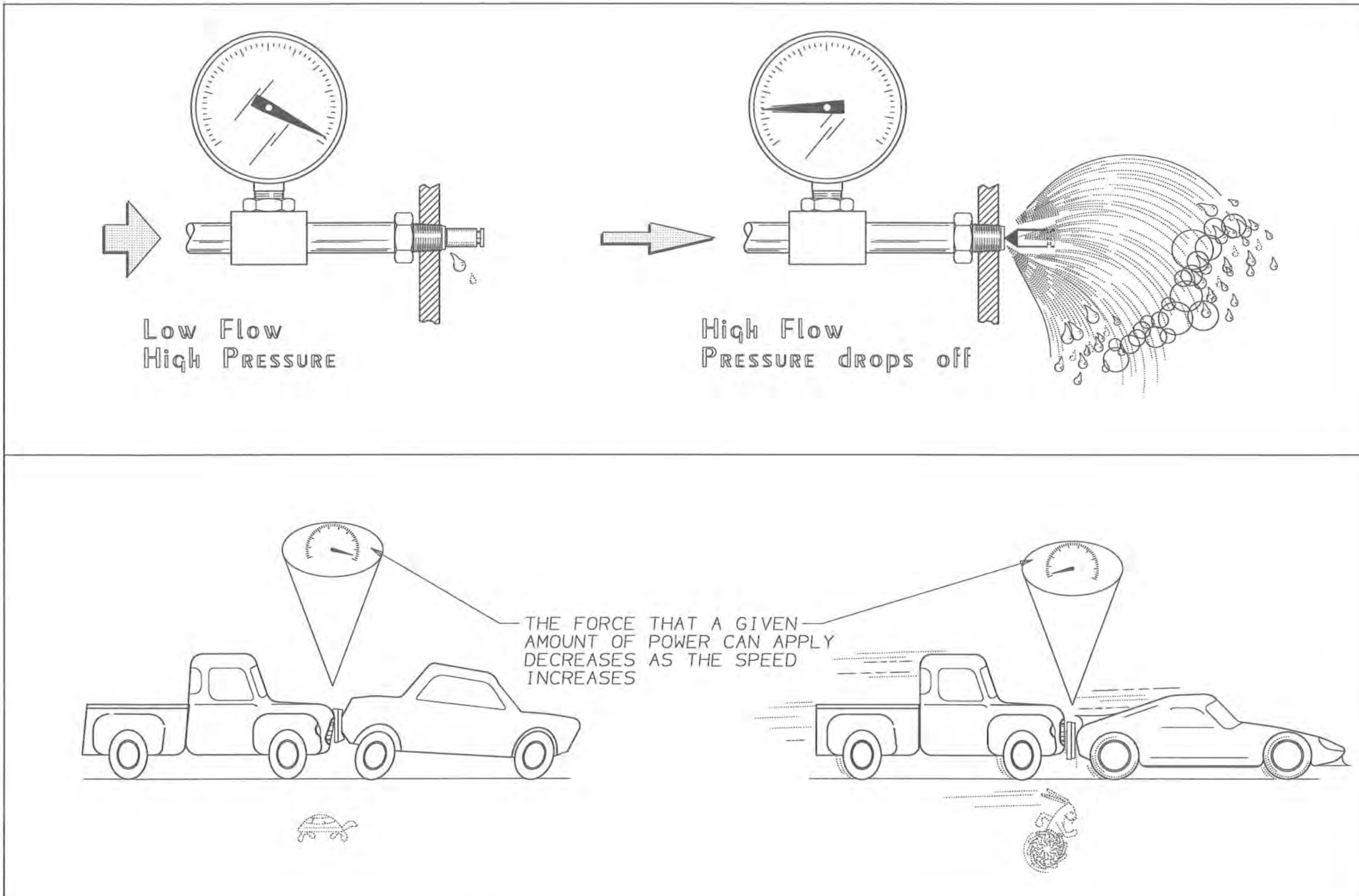


FIGURE 1 POWER IS THE RATE THAT FORCE IS OUTPUT

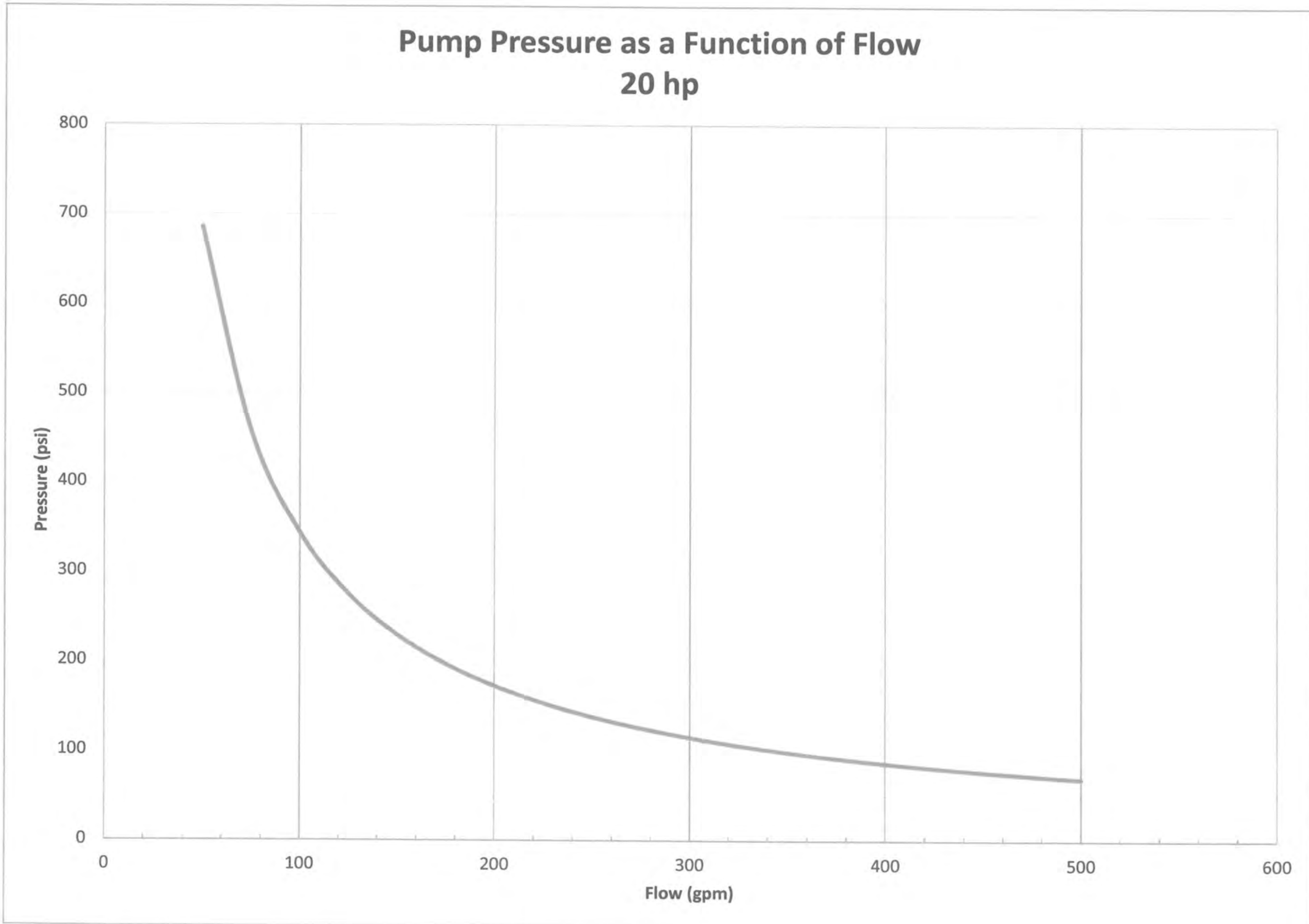

Pump Pressure as a Function of Flow
20 hp
Pressure (psi)
800
700
600
500
400
300
200
100
0
0
100
200
300
400
500
600
Flow (gpm)

Fig. 1a

Mathematical definitions of power (For the left side of your brain)

The technical definition of power is *rate of work.* Since work is force times distance and rate of distance is velocity, rate of work (power) boils down to is force times velocity (P=Fv). See the sidebar for a more in-depth discussion of the derivation of that formula. That goes along with our definition of power as the *rate* that force (torque) is applied. To tie that mathematical definition in with our conceptual example with the fuel pump let's visualize an idealized theoretical pump curve. (figure 1a) If our theoretical pump had the same power over its entire range of flows, the product of the pressure and the flow (P x V) is the same for any point on the curve and is equal to power (lbs./in^2 *times* in.^{3}/sec *equals* in.-lbs./sec). That should relate the technical definition of power as force times velocity (or pressure times flow) with the idea that a given amount of power will have higher force at lower speeds.

What all this means to the average performance enthusiast is that if you can find the force being applied at a certain velocity, you can find the horsepower. That's easy enough since a force acting on a mass causes an acceleration (which is what we're after in the first place). So to find force we measure acceleration then divide by the mass that was being accelerated. The trick here is that we can't measure acceleration at a certain velocity, so we measure it over a range, say 50 to 60 miles per hour. Dividing that acceleration by the mass that was accelerated would give us the force that is being applied at 55 mph.

Another definition of power is the *time rate of change of energy*. That allows us to find power directly if we know an initial and final energy state and the time it took to get from one to the other.

A vehicle's energy is a function of two things: its mass and the square of its velocity. Your Aunt Hilda gets on the freeway and three exits later finally has her Yugo XL up to 60 mph. Your racecar weighs the same as your auntie's Yugo, but it's doing 60 by the time it goes 120 feet down the track. Those two cars of equal mass travelling at the same velocity have the same energy. Dividing the energy by how long it took each car to get there yields power:

-equation marker-

Don't let the equations intimidate you. In this high-tech world the one who understands the physics of what's happening is going to see less tail lights. Hey, if winning were easy, everyone would be doing it. Besides, it's not that complicated. Examine this equation. The variables are mass, velocity and time. Since the mass of your car doesn't really change during that time all we have to measure is the change in velocity and the time it took to occur. You'll recall that our name for that is *acceleration.* Since the only way to accelerate a mass is with a force, power is proportional to force (or torque). So while it's technically correct to say that torque accelerates, horsepower does have a component of torque, that is force.

If you really want to sort out the math on this analyze the power from energy equation you'll notice that what we're solving is really velocity *squared* divided by time but acceleration is velocity only divided by time. To reconcile that notice that one of the velocity terms can be coupled with the mass and time term to give us a force (the 1/2 comes from taking the derivative of power to get energy, but you don't want to get into that.) That force (that causes the acceleration v/t) multiplied by the velocity left over from the v^2 term gives us the same definition we arrived at using the power equals rate of work definition (P=Fv).

Horsepower vs. torque

Now that we have a feel for power the physical quantity, let's examine horsepower in particular. When I was a kid I had a job driving a tractor. One day, it came up in conversation with my boss that the John Deere I was driving had a 90 hp engine. That seemed really low to me, as cars at that time were coming from the factory with 350 hp engines. Then I realized what the deal was. "But the gearbox multiplies that horsepower, right?" Wrong, my boss said, and I went away thinking my boss didn't know anything about horsepower. The flaw in my thinking is obvious. The gearbox multiplies *torque* but *divides* speed, so higher torque is put out at a lower rate resulting in the same power (disregarding friction). Remember the relationship of torque to power: power is the *rate* that torque is output. The equation looks like this.

-equation marker-

The derivation of how we got from P=Fv to this monster is described in the sidebar. What the equation illustrates is that how much power is being output depends on where on the RPM range how much torque is being developed. Horsepower is torque multiplied by a number which depends on the RPM. At 5252 RPM that number is 1.00, which means that the horsepower at that point exactly equals the number of ft.-lbs. of torque. Always. Below that RPM, the *number of ft-lbs* of torque will be higher than the horsepower and above that RPM the horsepower will be higher than that number.

That's where I was always confused. All this talk about torque and horsepower had me thinking that you could build either torque or horsepower at a certain point on the RPM band, which is not true, of course. For any torque amount at a given RPM there is only **one** horsepower value. When you change the torque anywhere, you alter the horsepower according to that relationship (which you can't change).

Figure 2 shows a flat torque curve and one where the torque is a straight line with a positive slope. The top graph is simply an illustration of the multiplier which relates torque to horsepower. Here's a clue to the relationship of horsepower to torque based on RPM. Notice that the as the torque is constant the horsepower increases in a straight line and as the torque increases in a straight line the horsepower curve bends upward. That means that a 10 ft.-lb. increase in torque on the upper end of the curve yields a larger horsepower gain than the same increase on the low end. That's borne out in the next set of graphs. The three torque curves in **figure 3** are identical except for being shifted. The difference in the horsepower curves emphasizes the aspect of power as a **measurement** of rate. Not only do the shifted torque curves show higher horsepower figures for the same amount of torque, but the rate the horsepower increases for a given torque increase is higher, just as we would expect.

Net force and acceleration

So what's the role of power? It's useful to think of power as a measurement—a number or an indication. The statement that torque not power accelerates is 100% true in physics books, but

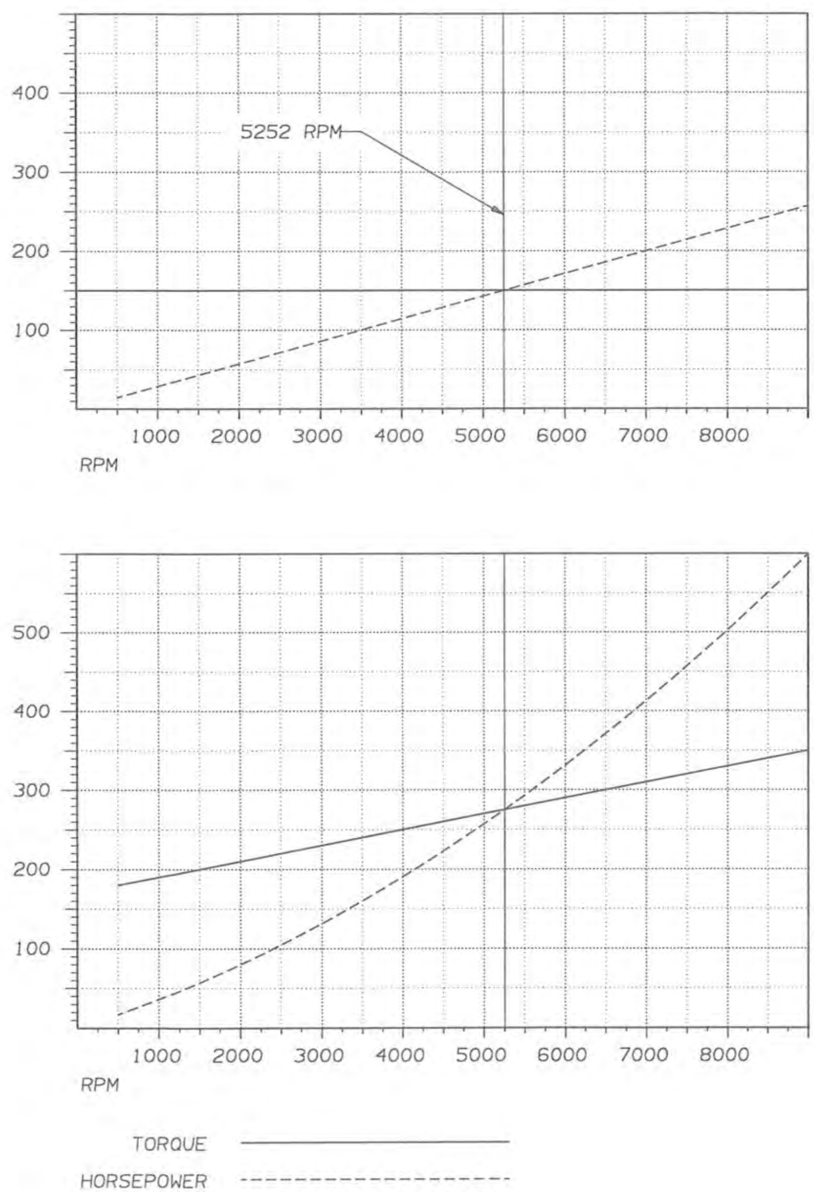


FIGURE 2 RELATIONSHIP OF TORQUE AND HORSEPOWER

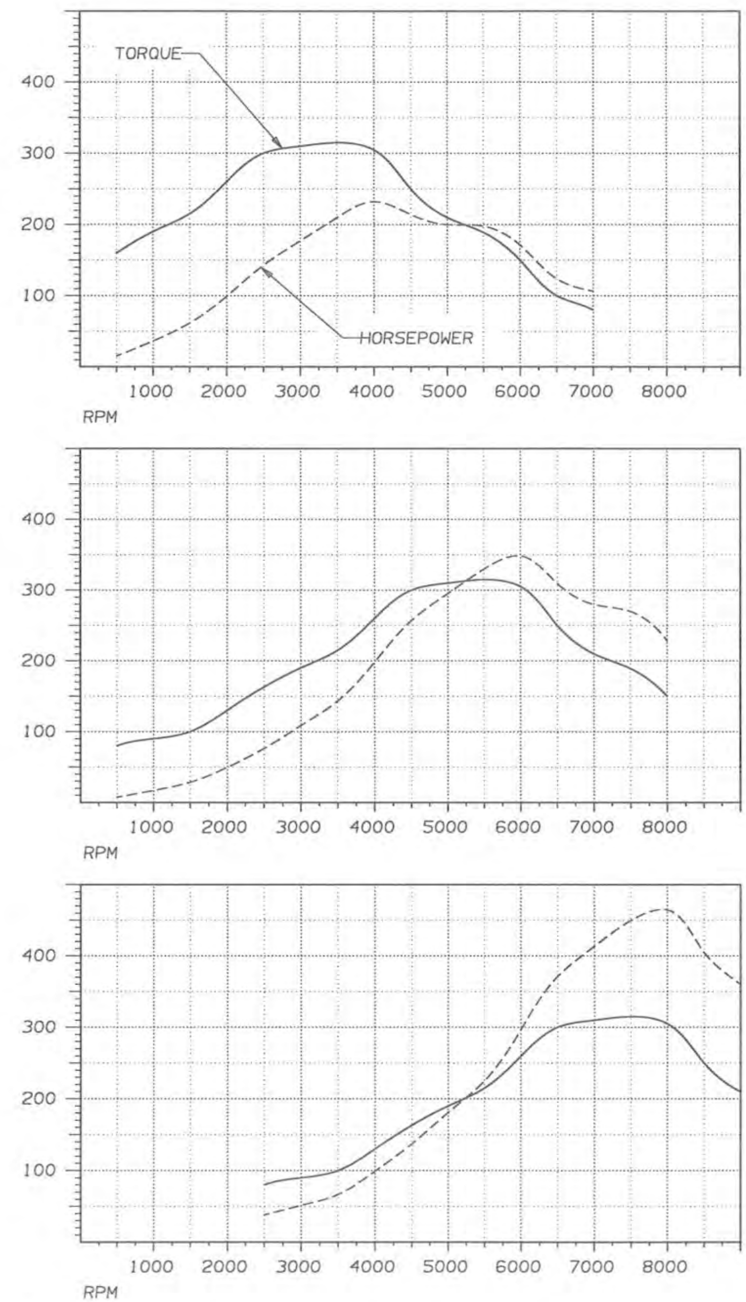


FIGURE 3 EFFECT OF SHIFTING THE TORQUE CURVE

what accelerates a racecar in the real world is net force. That is the sum of the forces making the vehicle accelerate minus the forces resisting that acceleration. At higher speeds, the forces resisting the car's motion are higher (**Figure 4**). **Figure 5** illustrates how these forces have the effect of bending the velocity curve downward by reducing the net effect of a constant force as speed increases. Higher horsepower figures (shifting the torque curve up on the RPM range) indicates that the vehicle will be capable of developing more force or pushing air at faster speeds. So is horsepower not important for acceleration at lower speeds? Think that one over in the context of what you now know about force and power. We don't want to give away all the secrets in one article.

The Paper Dyno

Now that we've figured out what power is, let's move on to determining how much you're getting out of your vehicle. If you have access to a chassis dyno, put this article away, you're done. Some garages advertise they have a dyno, but, as it turns out, the dynamometer down at your local tune-'em-up shop is usually nothing more than a roller that simulates the road resistance of a highway. So I'm going to show you a way to make use of the handy definition of power as rate of change of energy to draw a torque curve for your engine. To do that we're going to make some timed acceleration runs and record the results.

I've provided a worksheet called The Paper Dyno to plan, record and calculate your data. You'll want to make a photocopies of the original so you can scrawl freely away. Remember that this document wasn't found under a burning bush—modify it any way you want to make it more useful to you. Follow along on the chart while you read through the explanation of how to do this. It's really easier done than said. Basically, you're going to time how long it takes your car to accelerate a certain amount. Since acceleration is the result of a force acting on a mass, you'll be able to calculate the force that caused that. Then you'll correct for resistance and convert to torque which you'll convert to horsepower. There is also an equation for using initial and final energy to calculate horsepower directly, which you can then convert to torque. Either way you use, the

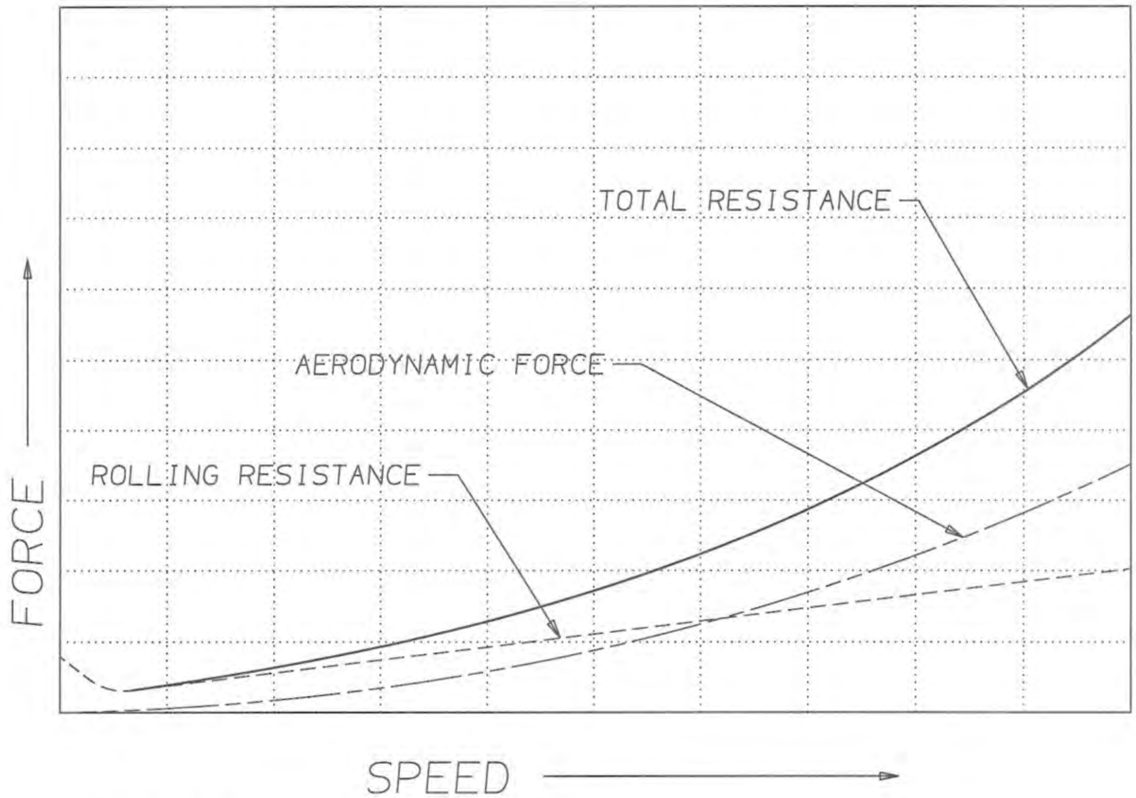


FIGURE 4: Forces resisting a vehicle's motion

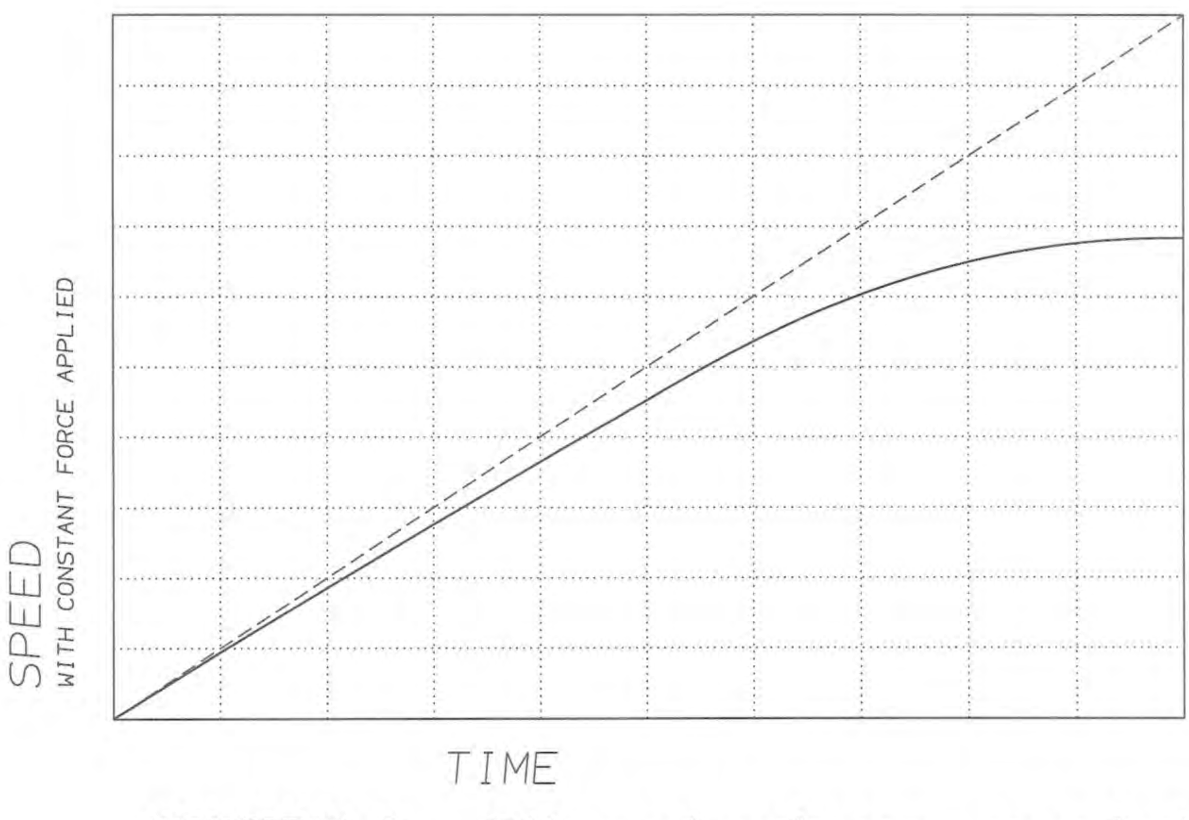


FIGURE 5: Effect of resistance on velocity given a constant applied force

The Paper Dyno ____________

WEIGHT	LBS		
THRUST INDEX "I"			
TIRE DIA D=	IN.		
REAR END R_R=	:1		

GEAR	RATIO	I=24R/D
1		
2		
3		
4		

EFPA
k
$= \frac{400\ F_d}{v^2}$

Column No.

	SETUP					TRIAL RUNS					CALCULATIONS		
Column No.	1	2	3	4	5		6	7	8	9	10	11	12
	GEAR	SPEED 1	SPEED 2	ΔV	V_{avg}		THRUST Δt ACCELERATION	THRUST $\frac{W\Delta V}{21.95\Delta t}$	DRAG Δt COAST DOWN	DRAG $\frac{W\Delta V}{21.95\Delta t}$	CORR. THRUST $F = F_a + F_d$	TORQUE $T=F/I$ $=5250P/N$	HP $P=TN/5250$ $=FV/375$
1		RPM	RPM			1.	SEC	LBS	SEC	LBS	LBS	FT-LBS	HP
		MPH	MPH	MPH	MPH	2.							
	NOTES:					3.							
2						1.							
						2.							
						3.							
3						1.							
						2.							
						3.							
4						1.							
						2.							
						3.							
5						1.							
						2.							
						3.							
6						1.							
						2.							
						3.							
7						1.							
						2.							
						3.							
8						1.							
						2.							
						3.							

RPM $N = 141v$
SPEED $V = N/141$

$a = \frac{\Delta V}{21.95\Delta t}$ $F_d = \frac{kv^2}{400}$ $k = \frac{18.22w\ \Delta v}{v^2\Delta t}$ $P = \frac{W(V_2^2 - V_1^2)}{16466\ \Delta t}$

numbers come out the same.

The first thing we need to know is how much mass we're accelerating. Most truck stops or metal salvage yards will let you use their scales for a small fee. Or just go sell that scrap engine block that's been kicking around your shop for so long. The scrap yard has to weigh your car before and after you unload it, so just ask to keep the weigh slip when you're done. If you haven't made a lot of modifications that would affect your car's weight, you can use its original ballpark weight from the blue book as a last resort. Write the weight you come up with in the appropriate box in the upper right hand corner.

At this point we need to introduce a concept called the thrust index which relates your engine torque to the force the road sees. If you wanted to argue with your buddy Dave, *thrust* is what actually accelerates a vehicle. The physical quantity is called force, and you already know how it's related to torque. A force applied to the end of a wrench times the length of the wrench is the torque output to the bolt (**Figure 6**).

To find the force from a torque, we divide the torque by the length of the moment arm it acts on, in this case, the radius of the tire. What the thrust index ("I") represents is the effective final drive ratio of your car. It's equal to the final drive ratio times 24 divided by your tire diameter.

-equation marker-

That's the same thing as your rear end gears corrected by the ratio of your tire's diameter to a "standard" diameter of two feet. What that allows you to do is find the thrust of your car by multiplying the torque by the index or find the torque by dividing the thrust by the index. For example, a car with 3.73 gears and a 26 inch rear tire would have a thrust index of 3.44:1. In a gear other than high (1:1) multiply the rear end ratio by the gear ratio to get the specific thrust index.

Knowing this index also makes it easy to calculate speed. Just divide the engine speed in RPM by fourteen times the thrust index.

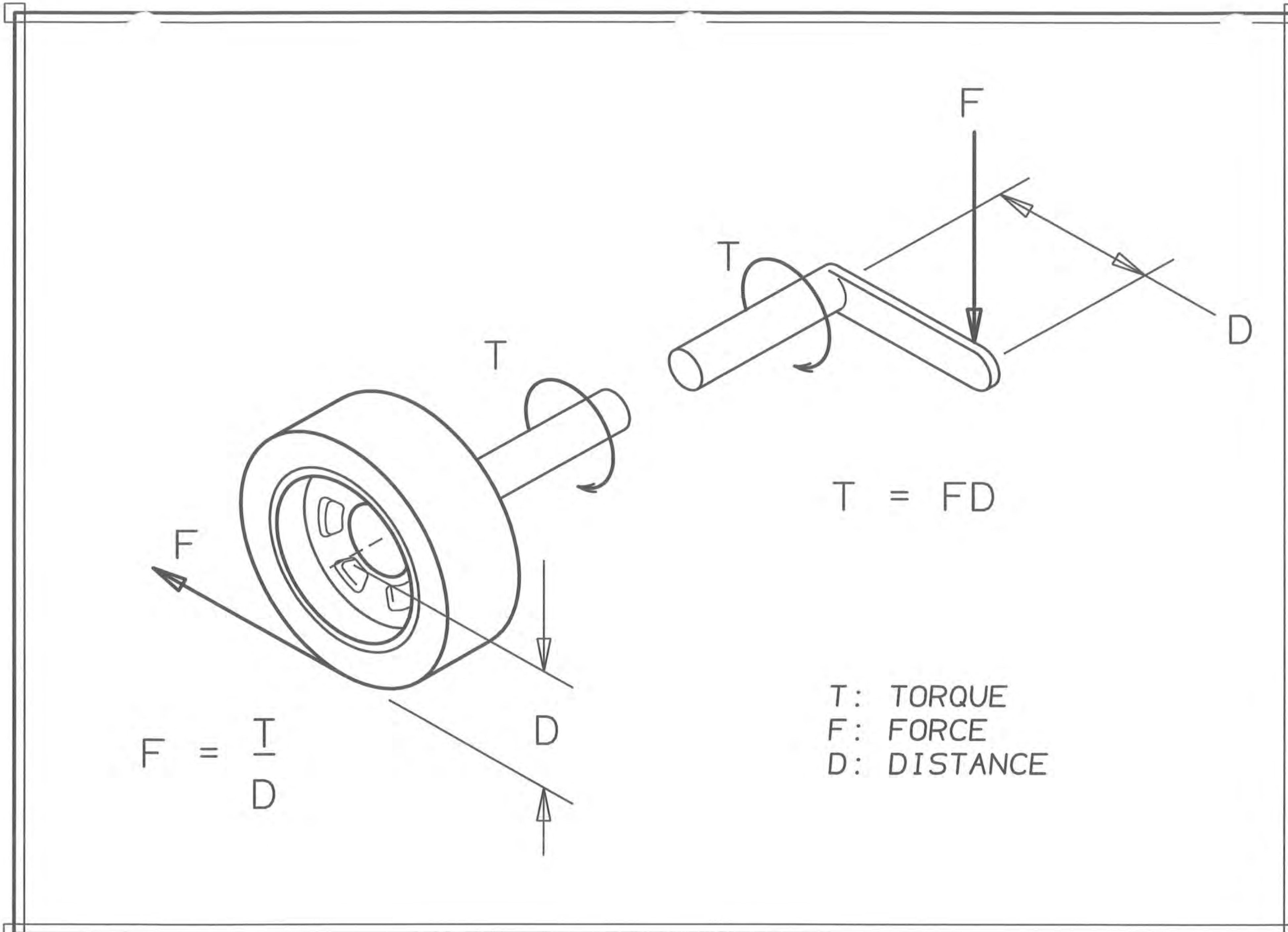
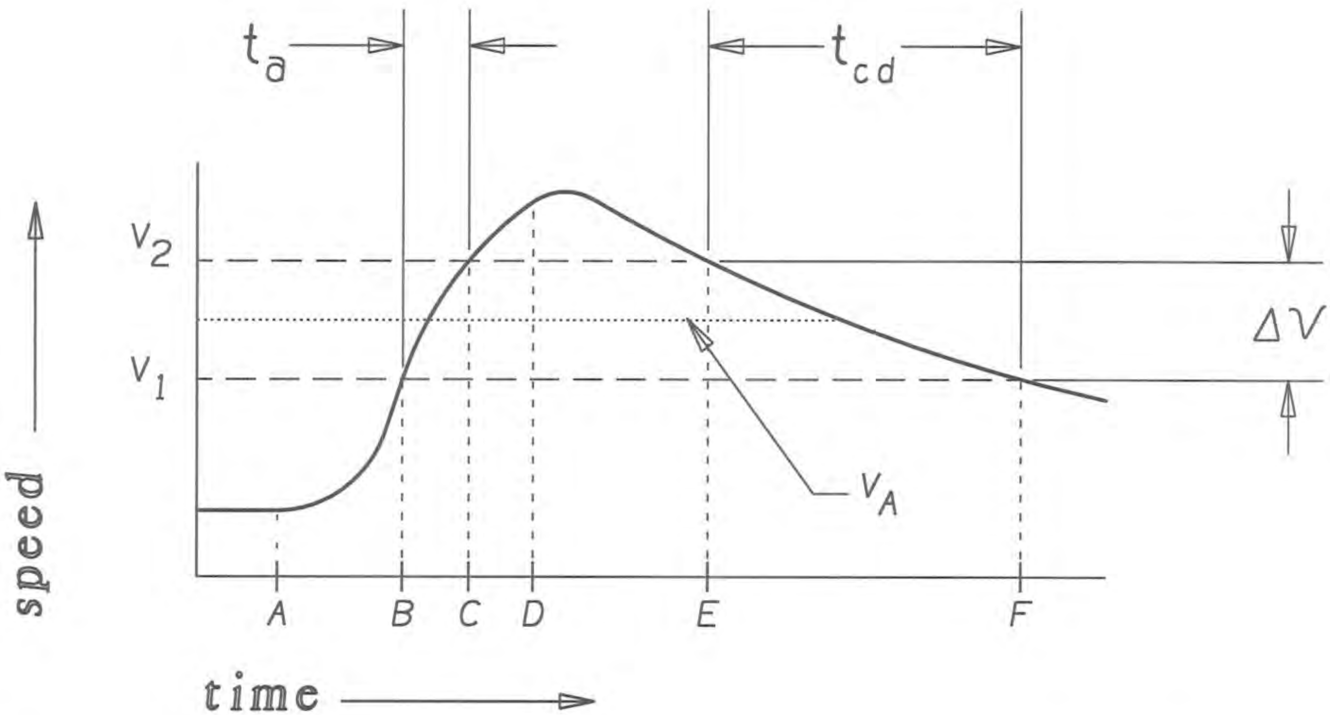


FIGURE 6: Relationship of TORQUE & FORCE

Calculate your thrust index and specific indices for the gears you will be using in the test and enter them in the boxes provided.

You need to calibrate your speedometer by timing it between mile markers or some other suitable way that would give you confidence in its accuracy. Alternatively, you may decide to use your tach and translate its readings into speed using "I" as indicated in the formulas on the worksheet. The difficulty with using the tach is that it doesn't give speed information on the coast down which is needed to calculate drag. Whichever way you choose, record the values you plan to use. What you want to do is cover enough of the RPM band that you can draw a complete curve from our data. You will probably have to run in a lower gear to get values for the higher RPM ranges. Running the tests at lower speeds will not only save on speeding tickets but has the added benefit of reducing the effect of aerodynamic and rolling resistance. The point is, plan out what you intend to do before you do it.

The spaces for speed and RPM are pretty self-explanatory. The formulas for relating the two using "I" are at the bottom of the form. Refer to **figure 7** for a good visualization of what you'll be doing. The space for Δv (delta v—*delta* being the highbrow word for difference) is just the speed range you'll be timing acceleration over and the v-average is just right smack in the middle of the high and low. Whenever a formula calls out *v* (as opposed to _v), it's this average of the span that's required. For example, if you're accelerating from 50 to 60 mph *v* (average) is 55 and Δv is 10 mph. When planning your ranges remember that a larger speed range reduces the effect of timing imprecisions: clicking the stopwatch exactly when the needle on the speedometer crosses the target speed. Unfortunately, it also flattens the real curve by averaging the acceleration over that range. What we're looking for is the instantaneous acceleration at the target velocity *v*, but since that's impossible to measure we use the average acceleration over the speed range _*v*. Luckily, there's a little calculus axiom that says the instantaneous slope of the curve somewhere in that range has to be the same as the average slope over the range. A span of about 10-20 mph might be a good starting place, depending on how screaming fast your car is and how fast you can



A. Full throttle
B. Stopwatch on
C. Stopwatch off
D. Off throttle
E. Stopwatch on
F. Stopwatch off

V_A Target (avg) speed
V_1 Low speed of test range
V_2 High speed of test range
ΔV Change in speed

t_c Timed interval of acceleration
t_{cd} Timed interval of coastdown

click a stopwatch.

With the form filled in to this point, let's go for a ride. You'll probably want a friend along to run the stopwatch. Just remember to include his or her weight in the calculations. You'll be doing multiple runs at each speed range and averaging the results to smooth out the variance. It's best to run trials for each condition in both directions to cancel effects of wind and road slope. Refer again to **Figure 7** which maps out what to do at this point. Start accelerating at full throttle a few mph below the low end of the selected speed range, start the timer exactly as you cross the low end, stop it exactly as you cross the top end, then get off the throttle a few mph after you stop the timer. Then, kick the car into neutral and time the deceleration over the same range.

After you record the results of your runs, it's time to do the calculations. Divide the acceleration ($\Delta v/\Delta t$) by 21.95 to convert acceleration in mph/s to acceleration in *g's*. Multiplying that by the weight gives thrust.

What you've got at this point are uncorrected values for the force your engine is applying to the road. This is where the data from the coast down runs will be used. The force the engine is transmitting to the road is reduced by the forces acting against the car. Since force accelerates (deceleration is acceleration with a negative magnitude) we can determine the forces resisting the car's motion by measuring the deceleration. The F_d (drag force) is determined the same way the acceleration force was, multiplying the acceleration in *g*s by the vehicle weight. That gives you the force that was resisting the acceleration which you will then **add** to the net thrust to find out the corrected thrust yield of your car. Now you can find the engine torque by simply dividing the thrust by the thrust index "*I*." Then horsepower is just the torque times the RPM divided by 5252.

It's safe to say you're going to be disappointed with the numbers you'll get from this method, but remember these are "where the rubber meets the road" numbers, not laboratory values. Quoting large horsepower numbers means nothing when you're looking at tail lights. The true value of this exercise is plotting the *shape* of the net torque curve. That will go a long ways toward understanding what your engine is doing and taking advantage of its strengths. The information you get is only as good as your planning and precision in measuring, but with proper execution it can be very useful. And it's a lot cheaper than going out and buying a dynamometer.

(*SIDEBAR*)

Let's get technical

Mathematical definitions of power

The technical definition of power is *rate of work* or work divided by time (W/t). Since we know that work is Force times distance (Fd) we can write the power equation like so:

-equation marker- $$P = \frac{Fd}{t}$$

Now, since we know that distance divided by time is velocity let's change the equation slightly to make it more useful to us.

-equation marker- $$P = F\frac{d}{t} = Fv \qquad \left(v = \frac{d}{t}\right)$$

Interesting. Power is equal to force times velocity. That goes along with our definition of power as the *rate* that force (torque) is applied. The equation P=Fv demonstrates that horsepower does have a torque component (F). What we're really pursing, after all, is acceleration, which occurs as a result of force acting against a mass and is defined simply as:

-equation marker- $$a = \frac{F}{m}$$

What we've done here is sort out the relationship of power to force (torque), which is the physical quantity that accelerates. So if we know the force (or torque) involved and the rate it's applied we know the power.

The rate that work is done is one definition of power. A physicist would say that power is the *time rate of change of energy*. So just for the fun of it, let's multiply power by time. That gives us energy. If you have an appliance that draws 100 kilowatts (power) and you run it for one hour you have 100 kilowatt hours of energy. Wait a minute. If you divide work by time to get power then *multiply* by time to get energy, you're back where you started. Are work and energy the same thing? Exactly so. That's called the work-energy theorem, which is just a high-brow handle for a handy principle that allows us to find power directly if we know an initial and final energy state and the time it took to get from one to the other (at an assumed constant rate).

Derivation of HP/Torque equation

We know that horsepower is the rate that torque is developed, but where does the equation

-equation marker-

come from? There's no voodoo involved here. All we're doing is changing the units we have into units we can use. This is done by multiplying by conversion factors and canceling units until we get the ones we want.

You're familiar with the general idea of converting units. For example, speed is distance divided by time, but speed in ft/sec doesn't mean a lot to us. So we divide the number of ft/sec by 5280 ft/mile and multiply by 60 secs/min then multiply again by 60 secs/hr. The seconds cancel each other as do the feet and the minutes, leaving us with miles divided by hour: *miles per hour*. We do the same sort of thing to derive the torque to horsepower equation. We just start with the definition of horsepower as the relationship of force and velocity and convert it into units we can use.

(1) $P = Fv$ General definition of power

(2) $$F = T\,\text{ft·lb}\; R \; \frac{2}{D\,\text{in}} \; \frac{12\,\text{in}}{\text{ft}} = \frac{24\,TR}{D}\ \text{lbs}$$

(3) $$v = \frac{V\,\text{rev}}{\text{min}} \; \frac{\text{wrev}}{R\,\text{rev}} \; \frac{\pi D\,\text{in}}{\text{wrev}} = \frac{\pi D V}{R}\ \frac{\text{in}}{\text{min}}$$

(4) $$P = \frac{24\,TR\,\text{lbs}}{D} \; \frac{\pi D V}{R} \; \frac{\text{in}}{\text{min}} \; \frac{\text{ft}}{12\,\text{in}} \; \frac{\text{sec hp}}{550\,\text{ft lbs}} \; \frac{\text{min}}{60\,\text{sec}}$$

$$= \frac{TV}{5252}\ \text{hp}$$

F from equation 1

v from equation 2

definition of horsepower

D: tire diameter in inches

T: torque in ft·lbs

V: engine speed in RPM

v: vehicle speed

F: force in lbs.

R: total final drive ratio

We start out with the definition of power as force times velocity (**step 1**). In the next two steps we calculate each of those terms (force and velocity) separately.

In **step 2** we determine the force (thrust) on the rear tires by starting out with the torque (T), multiplying it by the total gear ratio (R-which we'll later see drops out) to give us the torque to the rear wheel. Then we change that torque to force on the perimeter of the tire by dividing by the tire radius (D/2) then multiplying by 12 (*inches per foot*) so that we can cancel the foot units with those in *lbs-ft* of torque. The D term drops out later, but we need it now to straighten the units out. Cancel all the similar units in the numerator and denominator. The units we end up with are *pounds*, which are the units of force we were looking for.

We tackle velocity in **step 3**. Starting with engine RPM (V), we divide by the gear ratio (*engine revolutions per wheel revolution*) then multiply by *pi* and the tire diameter to get *inches per revolution*. That leaves us with *inches per minute*. We usually think of speed in terms of *miles per hour*, but we'll see in the next step that these units cancel more neatly.

Step 4 is pretty straightforward. We just multiply the terms we developed for force and velocity. That leaves us with units of *in-lbs per minute* which aren't too useful. So we divide by 12 *inches per foot* (cancel *inches*) then divide by 60 *seconds per minute* (*minutes* cancel) which would leave us with *ft-lbs per second*. Now we're getting closer. We know that there are 550 *ft-lbs per second* in a *horsepower*, so we divide by that to cancel everything else, multiply the numbers and, *voila!* we have *horsepower*.

A note about units

Let's clear up one final point. This particular numerical relationship between torque and power is valid only for the units of ft.-lbs and horsepower. I once heard someone (who should have known better) remark "Hmm. This car has more torque than horsepower. That's pretty good." Of course it's neither good nor bad. It simply means the measurements were both taken below 5252 RPM. You don't have <u>more</u> power than torque or <u>more</u> torque than power; you have a higher number of units one quantity than the other. The easiest way to make my belabored point is with an example. You can't say the distance to New York is more than the time in a day. You could say there are more *inches* between here and New York than *minutes* in a day. All this is an exercise in hair-splitting at best since you'll probably never hear power or torque described in other terms, but it's important towards understanding what the formula relating torque and force is really telling us.

The drag constant *k*

The trick is that the force in this case varies with the square of the velocity. What that means is that we have to find a way to predict the force at any speed once we've found the force at that particular speed. Finding that force is pretty simple once we know the drag constant "*k*."

What we're going to call the drag constant is actually the product of the drag coefficient and the frontal area of your vehicle. An engineer would call that the *effective flat plate area* which is the size of a flat plate that would offer the same resistance as your car if pushed through the air. Since the components that comprise the EFPA occur together we're going to take a shortcut and just call them *k*. To find this magical constant, apply the following formula:

-equation marker- $K = 400\, F_d / v^2$

The F_d (drag force) is determined the same way the acceleration force was, multiplying the acceleration in *g*s by the vehicle weight. Theoretically, the *k* value you get should be the same for each of those ranges. That's probably not going to happen, but we hope it's close. If the numbers always worked out exactly right, what would the challenge be? Racers would just get together with their calculators and whoever had the tallest stack of money would take home the trophy.

Now apply the *k* value to the average speed for each of the acceleration runs according to the formula for aerodynamic force:

-equation marker-

$$F_d = \frac{k v^2}{400}$$

Index of figures

5. Effect of resistance on acceleration

6. Relationship of torque and force

7. Snapshot of test run (*captioned*)